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Teaching mathematics in today's society: didactic paradigms, narratives and citizenship

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ABSTRACT

What is the paradigmatic direction of mathematical instruction? According to Yves Chevallard, father of the anthropological theory of the didactic, the current paradigm is characterized by an obsolescent form of monumentalism. But is a new paradigm possibly on the rise? And what is the role of powerful organizations such as the OECD? We reflect on these and related questions in connection to issues of citizenship, democracy, inclusion and standardization.

Introduction

What is the current paradigmatic direction of teaching and learning mathematics in an era which seems to be dominated by the normative influence of organizations such as the OECD? Yves Chevallard, father of the anthropological theory of the didactics, has proposed an influential analysis of the historical developments of mathematics education within institutions. In this account, the main paradigmatic shifts in the way mathematics is taught and learnt are seen as strongly tied to the political evolution of societies towards greater democratization and broader access to information and knowledge. As a result, according to Chevallard, the present paradigm of teaching mathematics may slowly give place to a new paradigm connected to emerging ideals of independence and critical citizenship.

In this article we first inscribe the discourses associated with Chevallard's envisioned paradigms within a classification of narratives offered by Ole Skovsmose. Second, we employ recent research in mathematics education (by Skovsmose, Paola Valero and Gelsa Knijnik) to analyze Chevallard's case for the emergence of the new paradigm. Third, we contrast the latter with historical and recent trends of curriculum development. In doing this, we further reflect on the apparent direction of

the practice of teaching mathematics as connected to issues of inclusion, citizenship, democracy and standardization. Our aim is to offer a critical, yet constructive perspective over Chevallard's historical periodization. The present theoretical contribution will thus hopefully serve in advancing the debate on the emergence and endurance of didactic paradigms of teaching mathematics within educational institutions.

Narratives and paradigms

Ole Skovsmose (2020) has discussed in this journal three main narratives associated with discourses on teaching and learning mathematics, themselves linked to the social role their proponents deem to be proper to ascribe to mathematics. The first narrative presents mathematics solely in terms of its inherent aesthetic and use-value characteristics and thus tends to consider mathematics educators as ambassadors of these sublime features towards the public. This is the *narrative of the sublime* (having an *aesthetic component* and a *use-value component*). The second narrative tends to look at mathematics education as suspect since it can be argued to be (more often than not) a tool for the governmentalization of people which may result in furthering their exploitation within the current political order, despite the good intentions of researchers and teachers. This is the *narrative of suspect*. The third narrative claims that mathematics education has a liberating potential as it may foster the development of critical citizenship by explicitly addressing political and social issues. This is the *narrative of critique*.

These three narratives are but a simplification of general positionings towards the discipline which often remain in the background of assumptions that tacitly guide research and practice in mathematics education worldwide. Indeed, the constraints implicit in the teaching of mathematics have often been of interest to proponents of the anthropological theory of the didactic introduced by Yves Chevallard (see Bosch & Gascón, 2014 for an exposition of the theory). Moving decades ago from Guy Brousseau's theory of didactic situations, Chevallard then concentrated his scholarship on the institutional and social dimensions of how mathematics is taught and learnt. In particular, Chevallard (2015)¹ has discussed the implicit didactic constraints bearing on the educational system by introducing the concept of didactic paradigm. In general, a *didactic paradigm* is, for Chevallard, a set of (often unspoken) rules which define the content to be taught/learnt within a didactic institution and the forms of teaching/learning in it. This notion was further defined and placed within the context of the anthropological theory of the didactic by Gascón and Nicolás (2019, pp. 44-45) in this journal.² Specifically, as to the mode of teaching and learning mathematics, Chevallard identifies two different historical paradigms:

¹ Drawn from Chevallard's address at 12th International Congress of Mathematics Education.

² The anthropological theory of the didactic concentrates on the study of didactic facts, defined as social situations in which at least one person tries to "learn" something (the didactic stakes), usually helped by some other person or group of persons. Notice that any didactic situation (as any other social situation) can be thought as a situation in which some constraints are applied, i.e., a set of insurmountable conditions which have to be taken as objective by the participants

- The most archaic, which we may call the *traditional paradigm*, is the paradigm which was organized around the study of doctrines or systems of great thinkers. As the most noteworthy example, Chevallard cites the fact that up to circa the 19th century people used to study Euclid's *Elements* as, for instance, historians of philosophy still study Hegel's *Phenomenology of Spirit*. This was the paradigm of "hailing and studying authorities and masterpieces" (Chevallard, 2015, p. 174).
- Progressively, from the 19th century onwards, the previous paradigm faded away, making way for the current paradigm of teaching mathematics, which Chevallard calls *the paradigm of visiting monuments*. As an example, Chevallard cites the case of Heron's formula for the area of the triangle which "is approached as a monument that stands on its own, which students are expected to admire and enjoy, even when they know next to nothing about its *raison d'être*, now or in the past" (p. 174).

The main difficulty with the current paradigm, according to Chevallard, is that it inclines people to perceive education as a highly institutionalized endeavor which is often self-referential and which has little or no connection with concrete world matters. Additionally, the current paradigm is not immune to reference to authority (of tradition or of experts) mandating which 'monuments' are or are not to be learnt. For Chevallard, the school curriculum has become nothing but an

epistemological 'monumentalism' in which knowledge comes in chunks and bits sanctified by tradition and whose supposed 'beauty' has been enhanced by the patina of age; that students have to visit, bow to, enjoy, have fun with and even 'love'. (p. 176)

In other words, there is no true justificatory reason for choosing this or that "monument" to study other than reference to tradition or to what experts deem to be worthy or useful: "the chief flaw in the paradigm of visiting monuments, which relates to the undemocratic ethos in which this paradigm originated, has to do with the choice of 'monuments' to visit at school" (p. 177). Nevertheless, Chevallard tends to be optimistic for the future and suggests that the paradigm of visiting monuments is slowly fading away, leaving room for a third yet-to-be-established paradigm:

- The *paradigm of questioning the world*, described by Chevallard as a counter-paradigm; as the potential polar opposite of the traditional paradigm. Within the new paradigm, as Chevallard envisions it, what will be valued most in education in general and in mathematics education in particular will not be the educational content to be known, but the fact that all people will be given the means to get to know what they want to know, according to their own interests and always instrumentally to the understanding of the real world (p. 177). The examples Chevallard explicitly directs his readers to are all linked to

and (at least for the time being) unmodifiable. The set of such conditions is the current didactic paradigm, i.e., the set of rules which implicitly or explicitly define what the didactic stakes are.

problems arising in connection to the study of scientific disciplines other than mathematics. For instance, one could be in the process of studying some physical or biological phenomenon and hence stumble across an equation having to do with one of them. At that moment one will ask oneself questions about that equation, trying to understand, for instance, where it comes from and what it means. (p. 178)

Of course, from the perspective of Skovsmose's analysis of narratives, all the three Chevallardian paradigms (in their discursive component) are instances of the narrative of the sublime since in none of them the political enters directly into the didactic arena. However, one may notice in this historical succession of paradigms a progressive shift of the importance attributed to the aesthetic value of mathematics towards a greater and greater importance attributed to its use-value in understanding the world. Moreover, even if Chevallard suggests directing attention only to problems related to curricular sciences, it may be imagined that (at least some) individuals or communities, independently directing their attention to the world, will choose to confront themselves with historical, social or even political problems.

Furthermore, Chevallard explicitly ties the historical succession of paradigms to the political realm by connecting it to changes in the level of democratization of the society we live in (p. 175, p. 177). Indeed, the paradigm of questioning the world, Chevallard states, will be established on the basis of strongly anti-authoritarian ideals of independence related to the cultivation of an ideal of citizenship as free as possible from the influence of tradition and experts' opinions on what is or is not worthy to be learnt. According to Chevallard, education may then come to be seen as a quest for knowledge, empowering every individual with the right to pose questions and give answers, with the minimal possible guidance from experts and, in any case, as independently as possible.

Chevallard is aware, of course, that a shift to this new paradigm would imply a shift in the perception of the whole educational enterprise. He condenses the essentials of this shift into three major requirements. First of all, what will be needed according to him is the understanding of education as a lifelong process not restricted to childhood and puberty. Second, it will be required a new pedagogical ethos (an Herbartian ethos, he calls it, after the philosopher and pedagogue Johann Herbart), essentially involving a receptive attitude towards unanswered questions. Third, a shift will be needed from a retrocognitive perspective to a procognitive perspective, meaning a shift from a cognitive attitude that leads one to refer preferentially and almost exclusively to knowledge already known (or known by experts), to a cognitive attitude which inclines one to behave as if knowledge was essentially still to discover and conquer—or to rediscover and conquer anew.

Inclusion and citizenship

Chevallard summarizes the aforementioned three conditions for the establishment of the new paradigm as a shift, from the usual notion of school to the Greek notion of σχολή (*scholē*). Such notion “originally designated spare time devoted to leisure, but [...] evolved to mean ‘studious

leisure', 'place for intellectual argument', and 'time for liberal studies'" (p. 180). Notice that the historical notion of σχολή, if interpreted within contemporary ethical criteria, has in our opinion a twofold nature. We feel here the necessity to discuss it with some of the healthy skepticism which is in the tradition of the narrative of suspect. On the one hand, the practice of σχολή in ancient Greece was tied to the peculiar slave society in which it flourished, where only free and wealthy men had the opportunity and the time to access education. On the other hand, it is without doubt that precisely because of the peculiar political freedom these ancient men enjoyed, they could also cultivate a notion of politics and of agency within the political realm which appears today more radical than that which any contemporary education reformer has ever envisioned.

The main difficulty in transferring this classical ideal to the contemporary world is, however, the profound asymmetry between the group of ancient wealthy men practicing σχολή (and thus being "citizens" in the full Greek sense) and the notion of citizenship as understood in today's society. Indeed, those ancient Greek citizens were in their times a tiny minority of "masters" lacking any preoccupation for their material subsistence and free to exercise their agency within a society that valued them and their opinions at the highest level. On the contrary, if σχολή is to be extended to everyone living today or in the future, doubts may be cast as to whether every individual (man, woman, wealthy or poor, i.e., all citizens in the contemporary sense) really could fully participate in such an educational framework. This worked in the past and presumably will work in the future only for those who are not excluded from the political and economic system in which they live.

Chevallard may be aware of these issues as he connects the persistence of the paradigm of visiting works with the not-completely-achieved democratization of society. Nevertheless, Chevallard seems to envision an historical tendency of society to become freer, more democratic and more Herbartian, a fact which he explicitly links to, among other things, the increasing availability of scientific information (previously stored in far-away libraries) to everyone possessing an internet connection, a fact which could potentially transform everyone into a "scholar".

Indeed, universal access to digital libraries is a key component of the globalized informational society towards which we are progressing at a fast pace. According to Skovsmose and Valero (2002), the transformation towards the informational society brings about two major contradictions which are often overlooked: the paradox of inclusion and the paradox of citizenship.

The *paradox of inclusion* refers to the fact that the current globalization model of social organization, which embraces universal access and inclusion as a stated principle, is also conducive to a deep exclusion of certain social sectors. The *paradox of citizenship* alludes to the fact that the learning society, claiming the need of relevant, meaningful education for current social challenges, at the same time reduces learning to a matter of necessity for adapting the individual to social demands. The paradox of citizenship concerns in particular [...] the development of general competencies for citizenship, especially the capacity to act critically in society, and in this way have an impact on it. (pp. 6-7).

Chevallard's proposal thus overlooks the possible issues of economic and social exclusion brought about or left untouched by the transition to the informational society. Chevallard's paradigm of questioning the world appears suitable to the needs of an ideal-typical classroom composed of first-world children or adults with plenty of free time to devote to rational enquiry and whose mundane needs are being taken care of.³ However, those who are being for various reasons already marginalized will presumably not be favored by the transition to the informational society.⁴ Nevertheless, at least for the non-excluded, it would seem to be possible to conceive an Herbartian pro-cognitive education directed to the critical investigation of the political dimension of the world, as we argued in the previous section. This feature, then, would prevent Chevallard's proposal from falling into Skovsmose and Valero's paradox of citizenship as it may include true critical citizenship elements addressed to a broader (but still limited) audience than in previous times, as today wider access to knowledge through the internet is available to the public.

Realistically, however, even if access to information could be made genuinely universal, this would not be nearly enough to extend the practice of *σχολή* to every citizen. The key issue here, if we follow Chevallard's reasoning, is really whether our society is increasing or decreasing in its level of democracy and economic justice so that everyone is able to enjoy the same freedom of action that the ancient masters did. Only this will, in turn, provide everyone with the opportunity to enjoy a real *σχολή* in the classical sense. Indeed, the critical question for the emergence and the broad establishment of the new paradigm of questioning the world envisioned by Chevallard and by the anthropological theory of the didactic is the following: are we really progressing towards a more democratic and free society and is the diffusion of technology pivotal to this progression? As it has been argued by Valero and Knijnik (2015) in this journal, it may be the case that, in general, the very inclusion of technology in formal education is (among others) itself a factor which tends to emphasize the social and economic distinctions between people, and which fosters the fabrication of the most apt to be governmentalized rather than acting as a liberating and democratizing device.

Democracy and standardization

Naturally, the question of whether our society is becoming more or less democratic would be too difficult to approach here. We ask those who are not as skeptical as we are about society's progressive democratization to look at the matter from another, more particular, perspective. What

³ A "prototype classroom" in the sense of Skovsmose (2006).

⁴ For example, very recent research investigating the effects of the COVID-19 pandemic seems to point to the fact that online teaching does not act as an equalizing factor between learners coming from uneven social backgrounds (Engzell, Frey & Verhagen, 2021).

is actually happening to education worldwide? Is education in general and mathematics education in particular actually approaching Chevallard's paradigm of questioning the world?

Of course, distinctions between paradigms are often blurred and history does not proceed as smoothly as theorists would perhaps hope. As a nationally situated example of this, the curriculum proposal by Anichini, Arzarello, Ciarrapico and Robutti (2003) for the teaching of mathematics at the turn of the millennium (endorsed officially by the Italian Mathematical Union and the Italian Ministry of Education) simultaneously contain traces of the two later Chevallardian paradigms: mathematics has to be studied for its cultural significance to our society as well as for its instrumental significance. Both of these dimensions are explicitly connected by Anichini and collaborators to the participation of citizens to a democratic society.

Mathematics education must contribute to the cultural formation of the citizen so that he or she may participate in the social life with awareness and critical ability. [...] The configuration of the school curriculum must take into consideration both the instrumental and the cultural function of mathematics [...] Both are essential for a balanced formation of students (Anichini *et al.*, 2003, p. 3, our translation)

Thus, this particular proposal amounts to a juxtaposition and integration of the two paradigms where both the cultural and instrumental dimensions co-exist and complement each other.

Nonetheless, in order to sketch a more general answer to the question above, we may supplement Chevallard's periodization with a finer-grained periodization of the history of teaching mathematics in late Modernity. As a schematic simplification we may say that, concerning Western Europe, the teaching of mathematics was always part of what we might call the "curriculum", with ups and downs in relevance, but never constituted a "subject" of particular importance in the overall education of the young (with the exception of specific technical apprenticeships such as accountancy, sailing or various types of military training) up until late-Modern times, characterized by the industrial revolution and the consolidation of nation-states which gradually took direct control of most educational systems. At that time, arguably, mathematics gained relevance by its instrumental connection with the applied sciences and hence, in the minds of many, with the technical innovations that were rapidly changing the face of the world. Again, as a simplification, we may say that the way of teaching mathematics slowly evolved from the teaching of Euclid to the teaching of theorem-monuments in Chevallard's sense, while mathematics itself gradually acquired relevance in connection with the technological revolution.

Valero (2017) has discussed this issue with examples taken from the history of schooling in France, Luxembourg, and Italy. During the course of the 19th century a tension persisted between the classical values subsumed by the traditional curriculum based on languages and the new technological values associated to the new curriculum based on mathematics and sciences. This tension has gradually faded away during the 20th century in favor of the new curriculum which imposed itself over the advanced Western world only after the second world war. Thus, mathematics and the sciences gained the prominence they came to possess in the past century (and which retain with

even more strength in the current informational society), largely in opposition to the classical values of which, not only at a linguistic level, Chevallard's usage of the term *σχολή* is reminiscent of.

Furthermore, the decline of the classical ideals of schooling is largely due to the significance that various actors (scholars, publicists, policymakers, etc.) came to attribute to the knowledge of science in connection with the making of the subjects best suited to live in our society. This fact appears, for instance, in the enormous corpus of documents produced in the second decade of the 20th century by various national and supranational organizations (of which the most influential certainly is the OECD) with the aim of justifying the demand for more mathematical or scientific education in connection with the employment of standardized tests. It seems clear that the next "advances" in education worldwide will be led primarily towards the direction pointed at by such studies.

Without addressing here the debate on the scientific, ethical or teleological foundations of the scholarship connected to such organizations' policies, we notice that the studies and tests that they mandate usually link people's levels of mathematical attainment to the manifestation in them of various attitudes, skills or virtues which are deemed to be relevant or useful at the individual or social level. Indeed, with respect to education, one of the main concerns of the OECD, repeated in various official documents written over the decades, is to make as many people as possible enter the highly specialized techno-scientific workforce needed by governments and corporations of the globalized economy in order to incentive economic growth (Valero, 2017, pp. 122-123) and to achieve prosperity (Andrade-Molina, 2017).⁵ Clearly such objectives have nothing to do with Chevallard's notion of *σχολή*, which is instead reminiscent of the classical Greco-Roman values of independence of the truth-seeking philosopher-scientist, and which is perhaps more similar to the ideal of teaching of the privileged few in classical or *ancien régime* societies, whose educational values were largely characterized by the lack of explicit concern for economic usefulness.

Additionally, as Valero has remarked, the OECD's policies and tests help in framing education within "a comparative logic which differentiates the individuals/nations who excel and are 'on the top', from those who need to be 'adjusted' to become normal and have success" (2017, p. 130). The result of this state of affairs will presumably be an ever-increasing standardization of the educational and moral features on which schools and educational institutions are based (see Lindblad, Pettersson & Popkewitz, 2018).

⁵ Notice that in one of the most recent OECD's documents, setting its policy framework for 2030, emphasis is given on preparing people "for jobs that have not yet been created" ([https://www.oecd.org/education/2030project/contact/E2030%20Position%20Paper%20\(05.04.2018\).pdf](https://www.oecd.org/education/2030project/contact/E2030%20Position%20Paper%20(05.04.2018).pdf), p. 22), possibly in connection with the recent dramatic rise in unemployment caused by automation and global economic crisis. Furthermore, the document highlights the fact that students' motivation should be "more than getting a good job and a high income; they will also need to care about the well-being of their friends and families, their communities and the planet" (p. 2). As to how this overall well-being could be achieved via (or perhaps only measured by) standardized psychometric tests we remain rather unsure.

Are we then really entering an era of progressive democratization of education, one in which every individual or community will have the power to conduct research by inquiring into real-world phenomena, as Chevallard suggests? On the contrary, it seems that we are progressing at a fast pace towards an age of standardization of educational features, as a result of the desire of most of the institutional actors involved in education to conform to the OECD's canons and values. Indeed, the nature of the mathematical content the OECD's studies measure and the nature of the individual virtues they assume to be desirable, being the same for all countries, cultures and individuals, are setting a unique global standard to which all educational institutions are being pushed to conform. In view of this, it seems almost a platitude to affirm that we are rather moving away from a system in which learners or communities of learners or even nations have the right to pose the questions they deem relevant to their own quest for autonomously understanding the world.⁶

Observe that, regarding mathematical content, the quantitative studies the OECD mandates invariably tend to measure mathematical achievement *per se* or else mathematical achievements in loose connection with the "real world" viable to be tested by word problems. Thus, a standard of ready-made mathematical content predisposed to be tested by closed questions is created and pressure is imposed on various educational actors to conform their teaching and testing to such a standard. Nothing is more distant from Chevallard's ideal of a mathematics serving the investigation of the world, with no "monumental" existence of its own.

Finally, this global standardization appears to happen through the guidance of some specific groups of scholars (e.g., statisticians, "big data" experts, psychometricians, and economists) while scholars of other disciplines as well as the main actors involved (local policy makers, teachers, parents and most importantly students) have little or no voice in the transformation.⁷ Therefore, it seems that the answer to the question we posed at the beginning of this section is negative: it appears that we are not progressing to a democratization of the educational endeavor nor to an age in which all people have the right to pose the questions they consider to be noteworthy or useful for developing their interests independently from what those who are in power deem to be the correct pedagogical standard.

⁶ Of course, under the "problem solving" educational model propagated by the OECD, it is reasonable to expect that at least some individuals and communities will be empowered with the right to work on the questions that are relevant for them. However, such an educational model seems to be also moving towards shaping those individuals and communities into posing the questions which are preemptively deemed to be the "right" questions. We thank one of the reviewers of this article for bringing up this point.

⁷ As suggested in a 2014 public letter from academics from around the world to Andreas Schleicher, director of the OECD's Programme for International Student Assessment. Online at: <https://www.theguardian.com/education/2014/may/06/oecd-pisa-tests-damaging-education-academics>.

Conclusion

Notice that the mainstream narrative concerning mathematics education propagated by the OECD (briefly characterized in the previous section) is also an instance of the narrative of the sublime, but one not concerned with aiding individuals and communities in their quest for understanding the real world nor with the development of classical values of individual independence but rather interested in producing standardized tests with which to measure people against authoritatively-mandated pedagogical standards. In short, both Chevallard's and the OECD's are narratives of the sublime because they do not contain any explicit reference to the political.⁸ In view of this, it could be perhaps the task of the narrative of suspect to reveal how the reality hiding behind the narrative subsuming the paradigm of questioning the world will actually strengthen the current political and pedagogical regime, perhaps in connection to Skovsmose and Valero's paradoxes of citizenship and inclusion discussed above.

Nevertheless, we believe that the paradigm of questioning the world implicitly promises a break with the current mainstream trend in education dominated by the OECD's influence. What Chevallard prefigures as the new paradigm of mathematics education is crucially not the learning of mathematics as an independent discipline tied to a global standardized curriculum, but instead the learning of mathematics as an instrumental discipline for understanding and inquiring the world as independently as possible. Furthermore, as we said above, even if Chevallard does not explicitly suggest it, it would seem to be possible within the paradigm of questioning the world, to direct these inquiries to non-standard objects which may (at least in some cases) have a political or social nature.

Thus, the paradigm of questioning the world has more than one encouraging feature which would allow it to be considered positively by those who are inclined to understand mathematics education through a critical lens. These are the disavowal of the aesthetic component of the narrative of the sublime, the criticism of standard curriculum mathematics based on "theorem-monuments" and the demand for an anti-authoritarian questioning of the world aimed at the development of independent critical citizenship. However, for these very reasons, the paradigm of questioning the world suggests an approach to the teaching and learning of mathematics which could hardly be inscribed within the framework proposed (or mandated) by the OECD.

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⁸ The two narratives however differ in what type of 'use-value' they give most importance to. We may roughly say that in the paradigm of questioning the world, attention is paid to the intrinsic value mathematics has in allowing people to understand the world scientifically, while the OECD has historically given most prominence to the extrinsic value of knowing mathematics as connected to the knowers' ability to join the workforce.