

## **VISUALIZATION AS VISION, IMAGINATION AND INTUITION: REFLECTIONS ON GRADUATE STUDENTS STRUGGLING WITH A VISUAL CONJECTURING PROBLEM**

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*In this paper we present some considerations on a case-study involving 25 graduate students in mathematics who were asked to conjecture the solution to a mathematical visualization problem. Surprisingly, no one of the subjects identified the correct solution and defended instead an incorrect one. We interpret the results within a fresh theoretical framework which classifies visualization skills into visual, imaginative and intuitive skills. We argue that the difficulties the students experienced may be ascribed to the considerable amount of imaginative effort needed in order to solve this particular problem (failure in imagination) as well as to the subjects' tendency to overgeneralize (false deduction derived from failure in intuition) together with (at least for a portion of the students) a difficulty in thinking out of the particular didactic contract they assumed to be in place. Finally, we conclude with a reflection on the institutional setting the subjects were immersed into and suggest directions for further investigations by linking psycho-pedagogical considerations on visualization with related university curriculum reform.*

### **INTRODUCTION, PRELIMINARY DEFINITIONS AND RESEARCH QUESTION**

Research in visualization within mathematics education originated in the work of Alan Bishop and was later carried out by various authors: see (Presmeg 2014) for a compendium and see also the report (Kadunz & Yerushalmy 2015) for a survey on recent advances in visualization. In this paper, we will follow the mainstream lineage of research developed by Abraham Arcavi and Norma Presmeg albeit explicitly stressing on some hopefully clarifying preliminary definitions inspired from the work of psychologist Efraim Fischbein as well as from the writings of mathematicians Felix Klein, David Hilbert and Henry Poincaré. These novel definitions, while allowing us to enclose the theoretical discourse concerning visualization within finer lines of demarcation, will also furnish us with the right vocabulary for framing the discussion of the case-study presented below.

*Vision* may be defined unambiguously as the faculty by which we directly see things which are there for us to see. On the other hand, *imagination* is the faculty by which we see what is not there to see (in mathematics this usually happens in connection with some properties one wants to prove or show). It may be divided into passive imagination (the act of representing to the mind something prompted to us from an outside source) and active imagination (the act of representing to the mind something not prompted from the outside). Furthermore, *intuition* is the faculty by which we generalize the properties that we see or imagine.

Finally, in accordance with the definition given in (Arcavi 2003) we define *visualization* in mathematics as all that concerns the faculties/properties/abilities above.